Advanced Level

COMBINED MATHEMATICS

Pure MATHEMATICS

INTEGRATION 1
අනුකලනය

රුවන් දුරශාන්ත

B.Sc (Hon's)
University of Colombo

අනුකලනය
INTEGRATION
2021-THEORY

Ruwan Darshana
B.Sc (Hon's)

සිද්ධාන්ත සමඟ සිද්ධාන්තමය ගැටළු

අනුකලනයේ මූලික නීති

බහුපද / පරිමේය ශ්‍රිතවල අනුකලන ක්‍රම

01. n යනු ඕනෑම තාත්විකයක් සහ $n \neq -1$ නම්

$$\int x^n \, dx = \frac{x^{n+1}}{n+1}$$

02. සෑම සිද්ධාන්ත ප්‍රකාශයකම x වෙනුවට x හා වෙනස් ශ්‍රිතයක් අඩංගු වී x ලෙස ගෙන අනුකලනය, x ලෙස දී ඇති ප්‍රකාශනයේ x වල සංයෝගයක් වේවායි.

$$\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{n+1} \times \frac{1}{a}$$

03. $\ln|\text{හරය}|$ සිද්ධාන්තය:-

$$\int \frac{\text{හරය අවකලනය}}{\text{හරය}} \, dx = \ln|\text{හරය}|$$

04. හරය රේඛීය සාධකයක් හා ලවය නියතයක් අඩංගු හාග:-

$$\int \frac{p}{ax+b} \text{ ආකාරය} \Rightarrow \ln|\text{හරය}| \text{ සිද්ධාන්තය}$$

H-W
02/06

01. (1) $\int x^7 dx$ (2) $\int x^5 dx$ (3) $\int x^{5/2} dx$ (4) $\int x^{1/2} dx$

(5) $\int \frac{1}{x^3} dx$ (6) $\int \frac{1}{x^4} dx$ (7) $\int \frac{1}{\sqrt{x}} dx$ (8) $\int 5x^3 dx$

(9) $\int 10x^2 dx$ (10) $\int 4\sqrt{x}\ dx$ (11) $\int \frac{8}{x^2} dx$ (12) $\int \frac{9}{x^3} dx$

02. (1) $\int (x^3 + x^2 + x + 1) dx$ (2) $\int (x^5 - x^2 - x) dx$

(3) $\int x\sqrt{x}\, dx$ (4) $\int (x+1)\,(x+5) dx$

(5) $\int (x^2 + 3)^3 dx$ (6) $\int \left(x^2 + 1 + \frac{1}{x^2}\right) dx$

(7) $\int \frac{x^3+4x^2+1}{x^2} dx$ (8) $\int \frac{x^3+4}{x^5} dx$

03. (1) $\int (3x+1)^5 dx$ (2) $\int (4x-7)^8 dx$ (3) $\int (1-5x)^3 dx$

(4) $\int \frac{1}{(9x+4)^3} dx$ (5) $\int \frac{1}{(1-7x)^2} dx$ (6) $\int \frac{1}{(x+2)^2} dx$

(7) $\int \frac{1}{(x-1)^3} dx$ (8) $\int \sqrt{4x+1}$ (9) $\int \sqrt{5-2x}\ dx$

(10) $\int \frac{1}{\sqrt{8x+1}} dx$ (11) $\int (5x+1)^{3/2}\, dx$ (12) $\int (2x-1)^{7/2}\, dx$

බහුපද / පරිමේය ශ්‍රිතවල අනුකලන ක්‍රම

04. (1) $\int \frac{1}{x} dx$ (2) $\int \frac{1}{5x+3} dx$ (3) $\int \frac{1}{3x-8} dx$

(4) $\int \frac{1}{1-5x} dx$ (5) $\int \frac{3}{7x+2} dx$ (6) $\int \frac{1}{ax+b} dx$

(7) $\int \frac{P}{ax+b} dx$ (8) $\int \frac{9}{2-3x} dx$ (9) $\int \frac{3}{6x+1} dx$

බහුපද / පරිමේය ශ්‍රිතවල අනුකලන ක්‍රම

(10) $\int \frac{1}{25x^2+1} dx$ (11) $\int \frac{1}{9x^2+10} dx$ (12) $\int \frac{1}{4x^2+7} dx$

බහුපද / පරිමේය ශ්‍රිතවල අනුකලන ක්‍රම

06. (1) $\int \frac{1}{x^2-9}dx$ (2) $\int \frac{1}{x^2-25}dx$ (3) $\int \frac{1}{x^2-1}dx$

(4) $\int \frac{1}{25x^2-1}dx$ (5) $\int \frac{1}{49x^2-25}dx$ (6) $\int \frac{1}{x^2-7}dx$

(7) $\int \frac{1}{x^2-13}dx$ (8) $\int \frac{1}{9-x^2}dx$ (9) $\int \frac{1}{1-x^2}dx$

බහුපද / පරිමේය ශ්‍රිතවල අනුකලන ක්‍රම

07. (1) $\int \frac{1}{x^2-4x+3}dx$ (2) $\int \frac{1}{(x^2-x-2)}dx$

(3) $\int \frac{1}{x^2-2x-8}dx$ (4) $\int \frac{1}{x^2-9}dx$

(5) $\int \int \frac{1}{(x+1)(x+3)}dx$ (6) $\int \frac{1}{(2x+1)(x-6)}dx$

08. (1) $\int \frac{1}{x^2-2x+1}dx$ (2) $\int \frac{1}{x^2+4x+4}dx$

(3) $\int \frac{1}{x^2-6x+9}dx$ (4) $\int \frac{1}{x^2-10x+25}dx$

(5) $\int \frac{1}{(x+3)^2}dx$ (6) $\int \frac{1}{(3x+1)^2}dx$

(7) $\int \frac{1}{9x^2-6x+1}dx$ (8) $\int \frac{1}{25x^2+10x+1}dx$

09. (1) $\int \frac{1}{x^2+4}dx$ (2) $\int \frac{1}{x^2+2}dx$

(3) $\int \frac{1}{x^2-2x+10}dx$ (4) $\int \frac{1}{x^2-2x+2}dx$

(5) $\int \frac{1}{x^2+4x+13}dx$ (6) $\int \frac{1}{x^2-x+1}dx$

(7) $\int \frac{1}{x^2-3x+5}dx$

බහුපද / පරිමේය ශ්‍රිතවල අනුකලන ක්‍රම

10. (1) $\int \frac{x-1}{x^2-4x-5}dx$ (2) $\int \frac{3x+5}{x^2-3x+2}dx$

(3) $\int \frac{2x-1}{x^2-5x+6}dx$ (4) $\int \frac{3x+1}{x^2+4x-12}dx$

11. (1) $\int \frac{2x+2}{x^2+2x-1}dx$ (2) $\int \frac{2x-1}{x^2-x+7}dx$ (3) $\int \frac{x+3}{x^2-2x+1}dx$

(4) $\int \frac{(5x-1)}{x^2-4x-13}dx$ (5) $\int \frac{x}{x^2-2x+5}dx$ (6) $\int \frac{(x-1)}{x^2+2x+10}dx$

(7) $\int \frac{3x+1}{4x^2+2x+3}dx$

බහුපද / පරිමේය ශ්‍රිතවල අනුකලන ක්‍රම

12. (1) $\int \frac{1}{(x-1)(x-2)(x-3)} dx$

(2) $\int \frac{1}{x^2+4x} dx$

(3) $\int \frac{3x+2}{x^3+x} dx$

(4) $\int \frac{1-7x}{x^3-x} dx$

(5) $\int \int \frac{1}{x^3+1} dx$

(6) $\int \frac{1}{(x^2-4)(x-2)} dx$

(7) $\int \frac{1}{x^3+x^2+x+1} dx$

(8) $\int \frac{1}{x^2+5x^2+1} dx$

(9) $\int \frac{3x-5}{(x+1)^2(x+2)} dx$

(10) $\int \frac{1-7x}{(x^2-4)^2} dx$

(11) $\int \frac{x^3+5x+1}{x^2-1} dx$

(12) $\int \frac{5x^2+6x+1}{x^2-4x} dx$

බහුපද / පරිමේය ශ්‍රිතවල අනුකලන ක්‍රම

13. (1) $\int \frac{x^3+5x+1}{(x+1)} dx$

(2) $\int \frac{x^4+3}{x^2+1} dx$

(3) $\int \frac{5x-1}{x+3} dx$

(4) $\int \frac{x^2+8}{x^2+x+1} dx$

(5) $\int \frac{5x^4-3x+1}{1+x^2} dx$

(6) $\int \frac{x^2+1}{x^2+8} dx$

14. (1) $\int \frac{x}{1+x} dx$

(2) $\int \frac{x+2}{x-3} dx$

(3) $\int \frac{5x-9}{6x+5} dx$

(4) $\int \frac{3x-7}{8x-1} dx$

(5) $\int \frac{7-9x}{3+5x} dx$

බහුපද / පරිමේය ශ්‍රිතවල අනුකලන ක්‍රම

(4) $\int \frac{5}{\sqrt{1-x^2}}\,dx$

(5) $\int \frac{9}{\sqrt{16-x^2}}\,dx$

(6) $\int \frac{1}{\sqrt{4-9x^2}}\,dx$

(7) $\int \frac{1}{\sqrt{1-25x^2}}\,dx$

(8) $\int \frac{3}{\sqrt{25-4x^2}}\,dx$

බහුපද / පරිමේය ශ්‍රිතවල අනුකලන ක්‍රම

16. (1) $\int \frac{1}{\sqrt{x^2+1}}dx$ (2) $\int \frac{1}{\sqrt{x^2+9}}dx$ (3) $\int \frac{1}{\sqrt{x^2+13}}dx$

(4) $\int \frac{4}{\sqrt{x^2+5}}dx$ (5) $\int \frac{9}{\sqrt{4x^2+1}}dx$ (6) $\int \frac{1}{\sqrt{9x^2+5}}dx$

17. (1) $\int \frac{dx}{\sqrt{9-x^2}}$ (2) $\int \frac{dx}{\sqrt{25-4x^2}}$ (3) $\int \frac{dx}{\sqrt{16-9x^2}}$

(4) $\int \frac{dx}{\sqrt{5+4x-x^2}}$ (5) $\int \frac{4dx}{\sqrt{3-2x-x^2}}$ (6) $\int \frac{2\,dx}{\sqrt{3+2x-x^2}}dx$

(7) $\int \frac{dx}{\sqrt{1-2x-x^2}}$ (8) $\int \frac{8dx}{\sqrt{9+6x-x^2}}$

18. (1) $\int \frac{dx}{\sqrt{x^2+9}}$ (2) $\int \frac{3dx}{\sqrt{4x^2+25}}$ (3) $\int \frac{dx}{\sqrt{x^2+2x+11}}$

(4) $\int \frac{dx}{\sqrt{2x^2-9}}$

බහුපද / පරිමේය ශ්‍රිතවල අනුකලන ක්‍රම

19. (1) $\int \frac{x+2}{\sqrt{x^2+4x+8}}\,dx$ (2) $\int \frac{x+1}{\sqrt{5+2x-x^2}}\,dx$ (3) $\int \frac{(2x+3)}{\sqrt{5+4x-x^2}}\,dx$

(4) $\int \frac{2x-3}{\sqrt{5+4x+x^2}}\,dx$

ත්‍රිකෝණමිතික ශ්‍රිතවල අනුකලන ක්‍රම

H.W. 02/07

20. (1) $\int \cos 3x\, dx$ (2) $\int \cos 4x\, dx$ (3) $\int \sin 2x\, dx$
(4) $\int \sin 4x\, dx$ (5) $\int \sec^2 2x\, dx$ (6) $\int \operatorname{cosec}^2 x\, dx$
(7) $\int \tan 2x\, dx$ (8) $\int \cot 3x\, dx$ (9) $\int \tan(x/2)\, dx$
(10) $\int \sec^2 4x\, dx$

21. (1) $\int \sin 5x \cos x\, dx$ (2) $\int \sin 7 \cos 7x \cos 3x\, x\, dx$
(3) $\int 5 \cos 8x \sin 2x\, dx$ (4) $\int 7 \sin 4x \sin 6x\, dx$
(5) $\int$ coos $10 \cos 2x\, dx$

22. (1) $\int \tan x\, dx$ (2) $\int \cot x\, dx$ (3) $\int \sec x\, dx$
(4) $\int cosec\, x\, dx$ (5) $\int \sec 2x\, dx$ (6) $\int \operatorname{cosec} 3x\, dx$

23. (1) $\int \sin^2 x\, dx$ (2) $\int \cos^2 x\, dx$ (3) $\int \cos^2 2x\, dx$
(4) $\int \sin^2 5x\, dx$ (5) $\int \cos^2 4x\, dx$

24. (1) $\int \sin^4 x\, dx$ (2) $\int \cos^4 x\, dx$ (3) $\int \sin^4 2x\, dx$
(2) $\int \cos^4 3x\, dx$

25. (1) $\int \sin^3 x\, dx$ (2) $\int \cos^3 x\, dx$ (3) $\int \sin^3 2x\, dx$
(2) $\int \cos^3 2x\, dx$

26. (1) $\int \sin^2 x \cos^2 x\, dx$ (2) $\int \cos 7x \cos 3x \cos 2x\, dx$
(3) $\int \sin^2 x \cos 2x\, dx$

සාතීය ශ්‍රිතවල / a^x ශ්‍රිතවල අනුකලන ක්‍රම

27. (1) $\int e^{2x}\,dx$ (2) $\int e^{5x}\,dx$ (3) $\int e^{3x+5}\,dx$

(4) $\int \frac{1}{e^{4x}}\,dx$ (5) $\int e^{2-7x}\,dx$

28. (1) $\int 2^{x}dx$ (2) $\int 5^{x}dx$ (3) $\int 2^{3x}dx$

(4) $\int a^{x}dx$ (5) $\int 3^{x+4}dx$ (6) $\int 4^{2-x}dx$

29. (1) $\int \cos 2x\,dx$ (2) $\int \sin 7x\,dx$ (3) $\int \sin 3x\,dx$

(4) $\int \cos^2 x\,dx$ (5) $\int \sin^2 x\,dx$ (6) $\int \sec^2 5x\,dx$

(7) $\int cose\,x \cot x\,dx$ (8) $\int cosec^2\,(3x+5)dx$ (9) $\int \sin^2 x\,dx$

(10) $\int \cos^2 x\,dx$ (11) $\int \cos^2 2x\,dx$ (12) $\int \sin^2 2x\,dx$

30. (1) $\int \tan x\,dx$ (2) $\int \cot x\,dx$ (3) $\int \sec x\,dx$

(4) $\int cose\,x\,dx$ (5) $\int \sin 7x\,\cos x\,dx$ (6) $\int \cos 10x\,\cos 4x\,dx$

(7) $\int \sin 9x\,\sin 13x\,dx$ (8) $\int \cos 9x\,\cos x\,dx$

ආදේශ මගින් අනුකලනය

31. (1) $\int \sqrt{25-x^2}\,dx$ (2) $\int \sqrt{1-x^2}\,dx$ (3) $\int \sqrt{4-x^2}\,dx$

(4) $\int \sqrt{3-x^2}\,dx$ (5) $\int \int \sqrt{5-x^2}\,dx$ (6) $\int \sqrt{9-4x^2}\,dx$

32. (1) $\int \sqrt{x^2+4}\,dx$ (2) $\int \sqrt{x^2+1}\,dx$ (3) $\int \sqrt{x^2+2}\,dx$

33. (1) $\int \sqrt{x^2-4}\,dx$ (2) $\int \sqrt{x^2-9}\,dx$ (3) $\int \sqrt{x^2-5}\,dx$

34. (1) $\int \frac{x^3}{1-x^2}\,dx$ (2) $\int \frac{x^3}{x^2+4}\,dx$

(3) $\int \sqrt[x]{\sqrt{4-x^2}}\,dx$ (4) $\int \frac{x}{x^2-1}\,dx$

35. 1) $\int (x+5)\sqrt{x+1}\,dx$ (2) $\int \frac{\sqrt{x-1}}{x+2}\,dx$ (3) $\int \frac{1}{(x+7)\sqrt{x-2}}\,dx$

(4) $\int \frac{1}{(x+7)\sqrt{x-2}}\,dx$

36. (1) $\int \frac{1}{1+x^{1/3}}\,dx$ (2) $\int \frac{1}{3+(x-1)^{1/3}}\,dx$ (1) $\int \frac{1}{2+x^{1/4}}\,dx$

37. (1) $\int \frac{1}{x^{1/2}+x^{1/4}}\,dx$ (2) $\int \frac{1}{x^{1/3}+x^6}\,dx$

38. (1) $\int \frac{dx}{(x+1)\sqrt{x^2-3}}$ (2) $\int \frac{dx}{x\sqrt{x^2+2x}}$ (3) $\int \frac{dx}{(x-1)\sqrt{x^2-2x}}$

(4) $\int \frac{dx}{(x-1)\sqrt{x^2+3x+1}}$ (5) $\int \frac{dx}{x(5x^2-4x+1}$

39. (1) $\int \sin^3 x\, dx$ (2) $\int \cos^3 x\, dx$ (3) $\int \sec x\, dx$

(4) $\int \cos^3 x \sin^2 x\, dx$ (5) $\int \sin^3 x \cos^3 x\, dx$ (6) $\int \cos 10x \cos 4x\, dx$

(7) $\int \cot^3 x\, dx$ (8) $\int \tan^3 x\, dx$

40. (1) $\int \frac{1}{1+\sin x}\, dx$ (2) $\int \frac{3}{2+\cos x}\, dx$ (3) $\int \sec x\, dx$

(4) $\int coses\, x\, dx$ (5) $\int \frac{1}{2-\cos x}\, dx$ (6) $\int \frac{dx}{3+5\cos x}\, dx$

(7) $\int \frac{dx}{3+2\sin x}$ (8) $\int \frac{dx}{4-5\sin x}\, dx$ (9) $\int \frac{dx}{\sin x+\cos x}\, dx$

(10) $\int \frac{dx}{4\cos x+3\sin x}$ (11) $\int \frac{dx}{3+2\sin x+\cos x}\, dx$ (12) $\int \frac{dx}{\cos 2x+3}$

(13) $\frac{dx}{\cos 2x+2\sin 2x}$ (14) $\frac{dx}{4-3\cos^2 x}$ (15) $\int \frac{dx}{\cos^2 x+4\sin^2 x}$

(16) $\frac{dx}{\sin x\,(1+\cos x)}$ (17) $\int \frac{3\sin x}{4+5\cos x}\, dx$ (18) $\frac{12\cos x}{5+13\sin x}$

(19) $\frac{1}{\sin^3 x}\, dx$ (20) $\int \frac{1}{\cos^3 x}\, dx$

41. (1) $\int \frac{\cos x}{\cos x+\sin x}\, dx$ (2) $\int \frac{2\sin x}{2\sin x+4\cos x}\, dx$

(3) $\int \frac{\sin x\, dx}{\cos x-2\sin x}\, dx$ (4) $\int \frac{5\cos x}{\sin x+3\cos x}\, dx$

(5) $\int \frac{\cos x-3\sin x}{2\sin x+4\cos x}$ (6) $\int \frac{\sin\theta-\cos\theta\, d\theta}{3\sin\theta+\cos\theta}$

42. (1) $\int \frac{x^3}{2+\sqrt{4-x^2}}\, dx$ (2) $\int \sqrt{\sin x}\cos x\, dx$ (3) $\int \frac{x}{\sqrt{1-x^4}}\, dx$

(4) $\int \cos x\,(\sin x)\cos x\, dx$ (5) $\int \frac{x^2}{\sqrt{2-3x^3}}\, dx$ (6) $\int \frac{x}{2x^4+55}\, dx$

(7) $\int \frac{x^2}{x^6+2x^3+3}\, dx$ (8) $\int \frac{dx}{x^3\sqrt{x^2-1}}$ (9) $\int \frac{1}{x(1+lnx)}\, dx$

කොටස් වශයෙන් අනුකලනය

43. (1) $\int xe^{4x}\,dx$ (2) $\int x\sin x\,dx$ (3) $\int x^2\,|nx\,dx$

(4) $\int x\,\tan^{-1}x\,dx$

44. (1) $\int x^2e^x\,dx$ (2) $\int x^2\cos x\,dx$ (3) $\int 3x^3e^{2x}\,dx$

(4) $\int e^x\cos x\,dx$ (5) $\int e^{4x}\sin x\,dx$ (6) $\int x^2\sin 5x\,dx$

(7) $\int e^{2x}\sin 3x\,dx$ (8) $\int \tan^{-1}x\,dx$ (9) $\int \sin^{-1}x\,dx$

(10) $\int In\,x\,dx$

45. (1) $\int x^2 \cos x \ dx$ (2) $\int 3x\, e^x dx$ (3) $\int \sqrt{x^2}\, e^x dx$

(4) $\int x e^{2x} dx$ (5) $\int x^3 e^x dx$ (6) $\int x\, e^{ax} dx$

(7) $\int x \sin x \ dx$ (8) $\int x \cos x \, dx$

46. (1) $\int x^2 \cos x \ dx$ (2) $\int x^2 \sin x \ dx$ (3) $\int x \sin 2x \, dx$

(4) $\int x \cos 2x \ dx$ (5) $\int x \sin x \ dx$ (6) $\int x \cos^2 x \ dx$

47. (1) $\int \tan^{-1} x \, dx$ (2) $\int \ln x \, dx$ (3) $\int \sin^{-1} x \, dx$ (4) $\int \cos^{-1} x \, dx$

48. (1) $\int e^2 \sin x \ dx$ (2) $\int e^2 \cos x \ dx$ (3) $\int x \ln x \, dx$

(4) $\int x \tan^{-1} x \, dx$ (5) $\int x \sin^{-1} x \ dx$ (6) $\int e^{3x} \sin 3x \ dx$

49. (1) $\int (In\ x)^2 \ dx$ (2) $\int x^2 \ln x \ dx$ (3) $\int x^3 \ln 3x \, dx$

(4) $\int x^2 \tan^{-1} x \ dx$ (5) $\int x (In\ x)^2 dx$

50. (1) $\int e^x \sin 2x \ dx$ (2) $\int \cos(In\, x)\, dx$ (3) $\int \sin(In\, x)\, dx$

(4) $\int \sec^3 x \ dx$ (5) $\int cosec^3 x \, dx$

51. (1) $\int \frac{\sin^{-1} x}{\sqrt{1+x}} \ dx$ (2) $\int ten^{-1} (2x-1) dx$ (3) $\int x \cos nx \ dx$

(4) $\int x \cos nx \, dx$ (5) $\int \frac{\sin^{-1} x/2}{\sqrt{2-x}} dx$ (6) $\int x(1+x)^7 dx$

(7) $\int x \sec^2 x \ dx$ (8) $\int cosec^4 4 \, dx$ (9) $\int \sec^4 x \ dx$

නිශ්චිත අනුකලනය

52. (1) $\int_0^5 x^2\,dx$ (2) $\int_{-1}^2 x^3\,dx$ (3) $\int_0^5 \frac{1}{(x+2)}\,dx$

(4) $\int_2^7 \frac{1}{x+1}\,dx$ (5) $\int_0^1 \frac{1}{x^2+1}\,dx$ (6) $\int_0^{\pi/2} \sin^2 x\,dx$

(7) $\int_2^5 \frac{1}{x^3-x}\,dx$ (8) $\int_0^{\pi/2} \cos^2 x + \cos 4x\,dx$

(9) $\int_2^{\pi} \sin^3 x\,dx$ (10) $\int_1^5 \frac{1}{x^3+x}\,dx$

53. (1) $\int_0^2 \sqrt{4-x^2}\,dx$ (2) $\int_0^{\pi/2} \frac{1}{2-\cos x}\,dx$ (3) $\int_1^8 \frac{1}{2+x^{1/3}}\,dx$

(4) $\int_{\sqrt{2}}^2 \frac{1}{\sqrt[3]{x^2-1}}\,dx$ (5) $\int_1^5 \frac{3x+1}{\sqrt{4x+5}}\,dx$ (6) $\int_1^{16} \frac{1}{x^{1/2}+x^{1/4}}\,dx$

54. (1) $\int_0^1 \tan^{-1} x\,dx$ (2) $\int_0^1 \sin^{-1} x\,dx$ (3) $\int_1^5 x^2 \ln x\,dx$

(4) $\int_0^{\pi} x\cos 4x\,dx$ (5) $\int_2^7 \ln x\,dx$

55. (1) $\int_1^2 \frac{2x+1}{(2-x)(x^2+1)}\,dx$ (2) $\int_a^b \ln x\,dx$ (3) $\int_1^2 \frac{2x+1}{x(x^2+1)}\,dx$

56. (1) $\int_{\pi/6}^{\pi/2} \sin x \sin 2x\,dx$ (2) $\int_0^{\pi/2} x\sin x\,dx$ (3) $\int_0^{a} \ln x\,dx$

57. (1) $\int_a^3 \frac{dx}{3\cos x+4\sin x}$ (2) $\int_0^2 \frac{dx}{\sqrt{4-x^2}}$

58. (1) $\int_1^2 x^2\sqrt{x^3+1}\,ax$ (2) $\int_1^2 x^2 \ln 2x\,dx$ (3) $\int_0^{\pi} \frac{\cos x+\sin x}{\cos x+2\sin x}$

59. (1) $\int_2^3 x\cos^2 x\,ax$ (2) $\int_0^{\pi/4} \frac{dx}{1+\sin 2x}\,dx$ (3) $\int_0^{\pi/3} x\sin 3x\,dx$

60. (1) $\int_0^{\pi/2} \sin^3 x\,dx$ (2) $\int_0^4 \frac{dx}{x^{1/2}+x^{1/4}}\,dx$ (3) $\int_1^2 \frac{2x^3+x+1}{x(x^2+1)}$

61. (1) $\int_0^4 \frac{1}{x^{1/2}-x^{1/4}}\,dx$ (2) $\int_0^{\pi/4} x\sec x\,dx = a\ln b \div c$ නම් a,b,c අගයන්න.

62. (1) $\int_{-1}^2 x^2\sqrt{x^3+1}\,dx$ *(model)* (2) $\int_0^{\pi/3} x\sin 3x\,dx$ *(model)*

(3) $\int_0^5 \frac{x+2}{(x+1)(x+3)}\,dx$ *(model)*

63. (1) $\int_2^3 \frac{3x-2}{x^3-x^2}\,dx$ (2) $\int_1^2 x^2\sqrt{x^3+1}\,dx$ (3) $\int_1^2 \frac{dx}{\sin x+\cos x}$

64. (1) $\int \frac{dx}{x^2+3}$ (2) $\int \frac{x^2}{x^2+3}dx$ (3) $\int \frac{x^4}{x^2-3}dx$

65. (1) $\int \frac{dx}{\sqrt{4-x^2}}$ (2) $\int \frac{dx}{\sqrt{x^2+5}}$ (3) $\int \frac{x^4}{x^2-3x+5}$

66. (1) $\int \frac{dx}{\sqrt{2+3x-2x^2}}\,dx$ (2) $\int \cos 3x\,dx$ (3) $\int \frac{(x^2-1)^2}{3x}dx$

67. (1) $\int \frac{x+1}{x^2-2x+5}$ (2) $\int \frac{dx}{\sqrt{5-4x-x^2}}$ (3) $\int \frac{x}{3-x^4}dx$

68. (1) $\int sin^2 x/2\,dx$ (2) $\int \cos^2 5x\,dx$ (3) $\int \frac{2x-5}{x^2-5x-7}\,dx$

69. (1) $\int \left(\frac{3}{1+4x^2}+\frac{8}{\sqrt{1-x^2}}\right)dx$ (2) $\int w^{mx}+5x\,\frac{1}{e^{mx}}dx$ (3) $\sqrt[3]{7-4x}\,dx$

70. (1) $\int \frac{\sin x}{1+3\cos x}\,dx$ (2) $\int \frac{\cos x}{1-2\sin x}\,dx$ (3) $\int 3^{x^3}x^2dx$

71. (1) $\int e^{-x^2}x\,dx$ (2) $\int (e^x+e^{-x})^2dx$ (3) $\int \sqrt{1+3\sin\cos x}\,dx$

72. (1) $\int \frac{x+2}{x^3-x}\,dx$ (2) $\int \frac{x^3}{\sqrt{x^3-1}}dx$ (3) $\int \frac{dx}{p^2x^2-q^2}dx$

73. (1) $\int \frac{dx}{(px-q)^4}\,dx$ (2) $\int \frac{2x+1}{\sqrt{x^2-3x+1}}\,dx$ (3) $\int \sin 7x\ ccos\,x\,dx$

74. (1) $\int coos\,mx\cos nx\ dx$ (2) $\int \sin mx\ \sin nx\ dx$ (3) $\int \frac{\cos^3 x}{\sin^2 x}\,dx$

75. (1) $\int (1+2\cos x)^3\,dx$ (2) $\int \cos^2 x\ \sin^3 x\,dx$ (3) $\int \sin^3 x\cos^4 x\,dx$

76. (1) $\int \frac{dx}{\sin^3 x}$ (2) $\int \frac{dx}{\cos^3 x}$ (3) $\int \cos^4 x\,dx$

77. (1) $\int \frac{\sin^3 x+1}{\cos^2 x}\,dx$ (2) $\int \frac{dx}{x^3-8}$ (3) $\int \frac{2x^2+x+2}{(x+2)(x^2+x+1)}\,dx$

78. (1) $\int \frac{x^3+x+1}{x^4-8l}\,dx$ (2) $\int \frac{x^4}{x^4-16}dx$ (3) $\int \frac{x+1}{(x^2+1)(x^2+x+1)}dx$

79. (1) $\int \frac{dx}{(x+2)(\sqrt{x^2+2x})}$ (2) $\int \frac{\sin x}{\sin 3x}dx$ (3) $\int \frac{\cos x}{\cos 3x}dx$

80. (1) $\int \frac{dx}{(1-x^2)^{3/2}}$ (2) $\int \frac{\sqrt{x^2+2x}}{x^3}dx$ (3) $\int \frac{dx}{x\sqrt{x^3-1}}$

81. (1) $\int \frac{\sin^{-1}\sqrt{x}}{\sqrt{x}}$ (2) $\int In\,(x^3+1)\,dx$ (3) $\int \sqrt{x}\,In\,x\ dx$

82. (1) $\int e^x \sin^3 x\,dx$ (2) $\int \tan^{-1}(1/x)\,dx$ (3) $\int x^2\sqrt{4-x^2}\,dx$

83. (1) $\int \sqrt{3+2x-x^2}\,dx$ (2) $\int \frac{\cos x+\sin x}{\cos 2x}dx$ (3) $\int \frac{dx}{\sin x-\cos x}$

84. (1) $\int \frac{3x^2+2x+1}{(x+1)^2(x^2+1)}\,dx$ (2) $\int \frac{dx}{x^3+8}$ (3) $\int \frac{7x-15}{x^3+2x^2+5x}\,dx$

85. (1) $\int \frac{2-\sqrt{1-x^2}}{\sqrt{1-x^2}}\,dx$ (2) $\int 3x\,dx$ (3) $\int a^{2x-1}\,dx$

86. (1) $\int x^2\tan^{-1}x\ dx$ (2) $\int x^2\sin x\ dx$ (3) $\int \frac{dx}{(5x+7)^4}$

87. (1) $\int \frac{dx}{(x-3)^4}$ (2) $\int \frac{\sqrt{3x+5}}{x}\,dx$ (3) $\int \frac{\sqrt[6]{x}}{1+\sqrt[3]{x}}$

88. (1) $\int \frac{dx}{5+3\cos x+5\sin x}$ (2) $\int x^2 x/2 \cos^2 x/2\, dx$ (3) $\int \frac{\sin 2x}{\cos^2 - \sin^2 x-1}$

89. (1) $\int \frac{\cos^3 x}{\sin^2 x+\sin x}$ (2) $\int \frac{\cos^2 x}{\sin^4 x}\, dx$ (3) $\int \frac{\tan^{-1} x}{x^2}\, dx$

90. (1) $\int \frac{dx}{\sqrt{x+1}+\sqrt{x}}$ (2) $\int \frac{e^x+2}{e^{2x}+2}\, dx$ (3) $\int \frac{dx}{e^{3x}-e^x}\, dx$

91. (1) $\int \frac{x\, dx}{(x^2+2x+2)^3}$ (2) $\int \frac{dx}{x^4-x^2-2}\, dx$ (3) $\int \frac{x+1}{3\sqrt{(2x+1)}}\, dx$

92. (1) $\int \frac{x^2\, dx}{\sqrt{x^2+x}}$ (2) $\int \frac{x^3\, dx}{1+3\sqrt{x^4+1}}$ (3) $\int \frac{x-a}{x^3+xa^2}\, dx$

93. (1) $\int \frac{dx}{x^2+\sqrt{1+x^2}}$ (2) $\int \frac{dx}{x(1-x^3)}$ (3) $\int \frac{dx}{x\sqrt{x^2}-2x+5}$

94. (1) $\int \tan^4 x\, dx$ (2) $\int \tan(ax)dx$ (3) $\int \frac{e^{3x}}{e^{2x}-1}\, dx$

95. (1) $\int \frac{dx}{2\sin x+\sin 2x}$ (2) $\int \frac{\tan^{-1} x}{(1+x^2)}\, dx$ (3) $\int \frac{1}{x\sqrt{a}\sqrt{nx+b}}\, dx$

96. (1) $\int \cos^2 2x\, dx$ (2) $\int \sin^4 x\, dx$ (3) $\int \cos^3 2x\, dx$

(4) $\int \sin^2 x \cos 2x\, dx$ (5 $\int \frac{\cos^4 x}{\cos 2x}\, dx$

97. (1) $\int e^x \sin 4x\, dx$ (2) $\int e^{2x} \cos 3x\, dx$ (3) $\int x^2 e^{2x+3}\, dx$

98. (1) $\int \frac{1}{2+\sin x}\, dx$ (2) $\int \frac{dx}{(x^2-4)(x^2-2x-3)}$ (3) $\int \frac{x^2-2x}{(x-1)(x+1)}\, dx$

99. (1) $\int \frac{1}{x^{1/4}+x^{1/2}}\, dx$ (2) $\int \frac{1}{x^2\sqrt{9-x^2}}dx$ (3) $\int \frac{x^3}{x^2-1}\, dx$

100. (1) $\int x^3\sqrt{x^3}+1\, dx$ (2) $\int x \sin 3x\, dx$ (3) $\int \frac{x^2+1}{x^2-1}\, dx$

101. (1) $\int x^3 \sin x^2\, dx$ (2) $\int \sqrt{\frac{1-x}{1+x}}dx$ (3) $\int \frac{1}{2\cos^2 x+1}\, dx$

102. (1) $\int \frac{dx}{(x^3-x)}$ (2) $\int \frac{(x+1)}{x^3+3x^2+5x}\, dx$ (3) $\int x^8(In\, x)^2\, dx$

103. (1) $\int \frac{2x^3+x+1}{x(x^2+1)}\, dx$ (2) $\int x\cos^2 x\, dx$ (3) $\int \frac{1}{x^{1/2}-x^{1/4}}\, dx$

104. (1) $\int x[e^x+\tan^{-1} x]\, dx$ (2) $\int \frac{dx}{1+\sin 2x}\, dx$ (3) $\int \frac{dx}{\sqrt{ax+b}-\sqrt{ax+c}}\, dx$

105. (1) $\int \frac{\cos x+\sin x}{\cos x\, 2\sin x}\, dx$ (2) $\int \frac{1}{2\sin x+\cos x+3}\, dx$ (3) $\int \frac{x^2}{\sqrt{2x+1}}\, dx$

106. (1) $\int x\cos 2x\, dx$ (2) $\int \frac{x^2+x+1}{(x+1)(x^2+1)}\, dx$ (3) $\int \frac{dx}{x^2\sqrt{x^2-1}}\, dx$

107. (1) $\int x\sin^{-1} x\, dx$ (2) $\int \tan^3 x\, dx$ (3) $\int \frac{x^3+1}{x^2+1}\, dx$

108. (1) $\int \frac{x^3+5x^2+6x-3}{x^3+2x^2-x-2}\, dx$ (2) $\int \frac{dx}{(x+1)\sqrt{x^2+3x+1}}\, dx$ (3) $\int x^2 \cos^2 x\, dx$

109. (1) $\int \sqrt{\frac{1-x}{1+x}}\, dx$ (2) $\int \sqrt{\frac{1-x}{x}}\, dx$ (3) $\int \frac{1}{x^2}\tan^{-1}\left(\frac{1}{x^2}\right)\, dx$

110. (1) $\int \frac{x^3+2}{x^2+2}\, dx$ (2) $\int \frac{dx}{1+2\cos^2 x}$ (3) $\int sec^3 x\, dx$

111. (1) $\int \frac{\cos x}{\sin x+7}\, dx$ (2) $\int cosec^3 x\, dx$ (3) $\int \frac{dx}{\sin x-\cos x}$